9.1 notes

* What is inference? p

Conclusions about a population parameter using statistics. $(\hat{p})$

- Inference is based on…

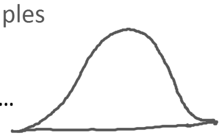
sampling distributions

- sampling distribution =

a histogram of lots of samples

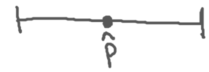
- Most sampling distributions are…

normal



Confidence Interval:

- FORM: statistic ± margin of error



- Example: Presidential Polls:

56% ± 3% $= (53\%, 59\%)$

56% ± 10% $= (46\%, 66\%)$

- Confidence Intervals are based on 3 things:

1) The sample proportion $\hat{p}$

2) The sample size

3) The confidence level

*** The confidence level gives us the...**

certainty that the true value is in the interval

*** Example: Presidential polls again**

65% ± 5% with 60% confidence

We are 60% confident that the true % of people approving of the president is btw 60% and 70%

65% ± 10% with 95% confidence

We are 95% confident that the true % of people approving of the president is btw 55% and 75%

65% ± 6% with 90% confidence

We are 90% confident that the true % of people approving of the president is btw 59% and 71%

Section 9.1: Confidence Intervals for a population proportion

- Parameter we are estimating: p Statistic we are using: $\hat{p}$

FORMULA: statistic ± margin of error

For a confidence interval of the population proportion

$$\hat{p} \pm Z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = (a, b)$$

- Z* is called the critical value
- 3 most common levels of confidence for Confidence Intervals and their Z*

90% confident $Z^* = 1.645$

95% confident $Z^* = 1.960$

99% confident $Z^* = 2.576$

Using the formula for a Confidence Interval, and the information below, complete the following examples.

EXAMPLE #1:

Find a 95% confidence interval when we have $\hat{p} = 0.15$, n = 400.

$$\hat{p} \pm Z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.15 \pm (1.96)\sqrt{\frac{(0.15)(0.85)}{400}}$$

$$0.15 \pm 0.035$$

$$(0.115, 0.185)$$

We are 95% confident that the true percent is btw. 11.5% and 18.5%

EXAMPLE #2:

Find a 90% confidence interval when we have $\hat{p} = 0.58$, n = 150.

$$0.58 \pm (1.645)\sqrt{\frac{(0.58)(0.42)}{150}}$$

$$0.58 \pm 0.0663$$

$$(0.5137, 0.6463)$$

EXAMPLE #3: We take a sample of 200 M&M's from a large bag and find that 32 of them are orange. Create a 90% confidence interval for the true proportion of orange M&M's in any bag.

$n=200$

$\hat{p}=\frac{32}{200}=0.16$

$C=90\%$

$z^*=1.645$

$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$0.16 \pm 1.645\sqrt{\frac{(0.16)(0.84)}{200}}$

0.16 ± 0.0426

$(0.1174, 0.2026)$

We are 90% confident that the true % of orange m&m's is btw. 11.74% and 20.26%.

CHANGE TO 95% CONFIDENCE

$0.16 \pm 1.96\sqrt{\frac{(0.16)(0.84)}{200}}$

0.16 ± 0.0508

$(0.1092, 0.2108)$

We are 95% confident that the true % of orange M&M's is btw. 10.92% and 21.08%.

$(0.1092, 0.2108)$

CHANGE TO 99% CONFIDENCE

$0.16 \pm 2.576\sqrt{\frac{(0.16)(0.84)}{200}}$

0.16 ± 0.0668

$(0.0932, 0.2268)$ 9.32%–22.68%

Conclusions:

- As the Confidence Level increases:

- The critical value
- The Margin of Error
- The Confidence Interval gets

Changing Sample Size Original: n=150 (0.5137, 0.6463)

Repeat Example #2 but use the sample size below. (90% confidence, $\hat{p}$ = 0.58)

n = 300

$0.58 \pm 1.645\sqrt{\frac{(0.58)(0.42)}{300}}$

0.58 ± 0.0469 $(0.5331, 0.6269)$

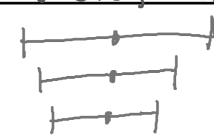
n = 600

$0.58 \pm 1.645\sqrt{\frac{(0.58)(0.42)}{600}}$

0.58 ± 0.0331 $(0.5469, 0.6131)$

Conclusions: As the Sample Size increases:

- The Margin of Error decrease
- The Confidence Interval gets narrower



Interpreting Confidence Intervals

(a, b)

Form: *(of the sentence interpretation)*

We are ____% confident that the true percent of __________ is between a and b %.

**Go back to example #3 and interpret your confidence interval.

% orange M&Ms (0.11736, 0.20264) 90% conf.

EXAMPLE #5: There is a poll of 50 American adults and they are asked what their favorite holiday is. 36% of them say Christmas. Create a 95% confidence interval for the percent of American adults who say Christmas is their favorite holiday. Interpret your interval.

$n=50$

$\hat{p}=0.36$

$C=95\%$

$0.36 \pm 1.96\sqrt{\frac{(0.36)(0.64)}{50}}$

0.36 ± 0.133

$(0.227, 0.493)$

We are 95% confident that the true % of Americans who say xmas is their fav. holiday is btw. 22.7% and 49.3%.

EXAMPLE #6: In a random sample of 3368 undergraduate students at a local college found that 2290 had a smart phone. Create a 95% confidence interval for the true proportion of undergrads that have a smart phone.

$n = 3368$

$\hat{p} = \frac{2290}{3368} = 0.68$

$C = 95\%$

$$0.68 \pm 1.96\sqrt{\frac{(0.68)(1-0.68)}{3368}}$$

$$0.68 \pm 0.0158$$

$$(0.6642, 0.6958)$$

We are 95% confident that the true % of undergrad students who own a smart phone is btw. 66.42% and 69.58%.

ADD TO NOTES:

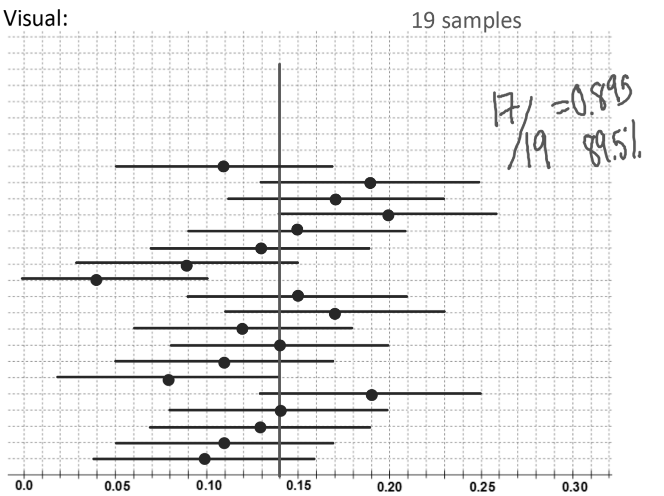
What does "95% confidence" really mean??

In repeated samples of size ______, 95% of the confidence intervals created will catch the true % of ____________.

Example: Orange M&Ms. Take a sample of 200.

In repeated samples of 200, 95% of the confidence intervals created will catch the true % of orange M&Ms.

Visual:



$17/19 = 0.895$

89.5%

WORKSHEET: Section 9.1 – Confidence Interval for Proportion

1. If 64% of a sample of 550 people leaving a shopping mall claims to have spent over $25, determine a 99% confidence interval estimate for the proportion of shopping mall customers who spend over $25. Interpret your interval.

2. In a random sample of machine parts, 18 out of 225 were found to have been damaged in shipment. Establish a 95% confidence interval estimate for the proportion of machine parts that are damaged in shipment. Interpret your interval.

3. A telephone survey of 1000 adults was taken shortly after the U.S. began bombing Iraq. If 832 voiced their support for this action. Create a 99% confidence interval and interpret the interval.

#3, 4, 6

4) An assembly line does a quality check by sampling 50 of its products. It finds that 16% of the parts are defective.

a. Create a 95% confidence interval for the percent of defective parts for the company and interpret this interval.

b. If we decreased the confidence level to 90% what would happen to:

i. the critical value?

ii. the margin of error?

iii. the confidence interval?

c. If the sample size were increased to 200, the same sample proportion were found, and we did a 95% confidence interval; what would happen to:

i. the critical value?

ii. the margin of error?

iii. the confidence interval?

5) A nationwide poll was taken of 1432 teenagers (ages 13-18). 630 of them said they have a TV in their room.

a. Create a 90% confidence interval for the proportion of all teenagers who have a TV in their room and interpret it.

b. What does "90% confidence" mean in this context?

c. If we increased the confidence level to 99% what would happen to:

i. the critical value?

ii. the margin of error?

iii. the confidence interval?

d. If the sample size were changed to 950, the same sample proportion were found, and we did a 90% confidence interval; what would happen to:

i. the critical value?

ii. the margin of error?

iii. the confidence interval?

6) Suppose a 90% confidence interval is stated as (0.3011, 0.4189).

a. What is the sample proportion from this sample?

b. What is the margin of error?

3) $0.832 \pm 2.576\sqrt{\dfrac{(0.832)(0.168)}{1000}}$

0.832 ± 0.0305

(0.8015, 0.8625)

We are 99% confident that the true percent of people who support the bombings in Iraq is between 80.15% and 86.25%.

4) (a) $0.16 \boxed{\pm 1.96\sqrt{\dfrac{(0.16)(0.84)}{50}}}$ margin of error

$0.16 \boxed{\pm 0.1016}$

(0.0584, 0.2616) = conf. interval

We are 95% confident that the true % of defective parts is between 5.84% and 26.16%.

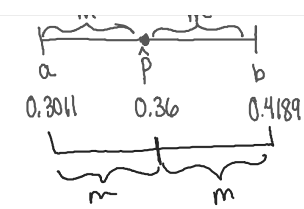
(b) i- decrease ii- decrease iii- narrower

(c) i- same ii- decrease iii- narrower

6) (0.3011, 0.4189)

(a) $\hat{p} = 0.36$

(b) m = 0.0589



HW: #2 + 5